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PACS. 25.75−q – Relativistic heavy-ion collisions.
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**Abstract.** – Rapidity spectra for protons at SIS-, AGS- and SPS-energies are analyzed within a Relativistic Diffusion Model. At low beam energies of 1.06 A GeV rapidity diffusion coefficients are related to the corresponding relaxation times via a weak-coupling dissipation-fluctuation theorem. At higher energies progressively larger deviations are found, rising to about an order of magnitude at SPS-energies.

In relativistic heavy-ion collisions the observed particle rapidity spectra are significantly broader than isotropic thermal distributions [1]. The discrepancy increases from SIS via AGS to SPS energies. The origin of the broad distributions has often been attributed to collective expansion of an equilibrated source. However, it has been suggested [2] that thermal equilibrium may not be attained during the short interaction time, and that the broad distributions could essentially be due to the nonequilibrium properties of the relativistic system.

Since phase-transition scenarios to a deconfined quark-gluon phase are based on the implicit assumption that at least a substantial part of the system thermalizes, it should be investigated carefully if this is actually the case, or whether the available data can be understood consistently with the help of nonequilibrium processes. In this work, the validity of a dissipation-fluctuation theorem in relativistic collisions is tested. Such a theorem is an important ingredient of a nonequilibrium- statistical description.

A very general description of relaxation processes in nonequilibrium systems is provided by the Fokker-Planck equation (FPE) [3]. In particular, it has been shown that proton rapidity distributions can be suitably described on the basis of a linear Fokker-Planck equation as outlined in the Relativistic Diffusion Model (RDM) [2]

$$\frac{\partial R(Y,t)}{\partial t} = \frac{1}{\tau_Y}\frac{\partial}{\partial Y}\Big[(Y-Y_{eq})R(Y,t)\Big] + D_Y\frac{\partial^2}{\partial Y^2}R(Y,t). \quad (1)$$

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The relaxation time $\tau_Y$ and the rapidity diffusion coefficient $D_Y$ determine the time evolution of the rapidity distribution. Rapidity spectra calculated from eq.(1) depend on $\tau_{int}/\tau_Y$ with the interaction time $\tau_{int}$, and $D_Y$. The interaction time may be taken from analyses of the data or from theoretical - preferably dynamical - models. Here I use impact-parameter dependent values from [4] which are proportional to the number of participants and have provided consistent descriptions of transverse energy distributions. These values are larger than freezeout-times from other analyses [1], but definite results are not yet available. However, future precise determinations of interaction times will simply lead to a rescaling of the transport coefficients. Hence, time rescaling does not affect the validity of the present analysis.

In a weak-coupling situation (where the duration of a single interaction is smaller than the time beween two subsequent collisions), the transport coefficients are related to each other through the temperature $T_p$ of the thermal equilibrium distribution. Whereas the usual form of the Einstein relation for momentum-like variables is $D_Y \sim T_p/\tau_Y$ in the theory of Brownian motion, the special form of the thermal equilibrium distribution in rapidity space induces a modified result. Even without detailed consideration of the microscopic structure of the system, this relation has been derived in [2] from the requirement that the stationary solution $R_{eq}$ of the FPE agree with the isotropic thermal equilibrium distribution $R_{th}$ in Y-space

$$D_Y^{th} = \frac{1}{2\pi\tau_Y}\left[c_p^T m_p^2 T_p\left(1+2\frac{T_p}{m_p}+2(\frac{T_p}{m_p})^2\right)\right]^{-2}\exp\left(\frac{2m_p}{T_p}\right). \tag{2}$$

For a given equilibrium distribution, the rapidity diffusion coefficient is then determined by the dissipative constant $\tau_Y$. The proton mass is $m_p$, and $c_p^T$ ensures that the corresponding thermal distribution function $R_{th}$ is normalized to 1 for each temperature $T_p$

$$\int_{-\infty}^{+\infty} R_{th}(Y)dY = 1 \tag{3}$$

with the normalized center-of-mass rapidity $Y = 2y_{cm}/y_1$ and the projectile rapidity $y_1$. This yields

$$c_p^T = \frac{y_1}{2m_p^2 T_p}\left[\int_{-\infty}^{+\infty}\left(1+2\chi_T(y)+2\chi_T(y)^2\right)\exp\left(\frac{-1}{\chi_T(y)}\right)dy\right]^{-1} \tag{4}$$

with

$$\chi_T(y) = \frac{T_p}{m_p\cosh(y)}. \tag{5}$$

At fixed incident rapidity $y_1$, the diffusion coefficient displays the expected behavior, namely, it rises with increasing temperature of the corresponding equilibrium distribution as in the Einstein relation; the rise is almost linear with $T_p$.

The dissipation-fluctuation theorem may, however, be violated in relativistic collisions where the coupling is strong and a large amount of energy resides in the interaction. Hence, I investigate in this work the validity of eq.(2) in relativistic systems from SIS to SPS energies.

For participant protons, $\delta$-function initial conditions lead to superposed Gaussian solutions of the FPE which - together with the corresponding interaction times for central collisions - provide analytical results for the rapidity spectra as outlined in [2]. Adjusting the rapidity relaxation time to the data and calculating the rapidity diffusion coefficient from eq.(2) with the temperature $T_p$ of the corresponding equilibrium distribution taken from other sources (such as hadron abundancies), model results for the rapidity density distributions are obtained.

Comparing these results to the data from SIS via AGS to SPS energies, I have tested the validity of the Einstein relations. At SIS-energies, the light Ni + Ni - system provides data in the range 1.06 - 1.93 A GeV [5]. With the weak-coupling value of the rapidity diffusion coefficient from eq.(2) the data are well reproduced at the lowest energy, as shown in the upper part of fig.1 (solid curve). Here, the dashed curve is the thermal equilibrium distribution with a temperature of 79 MeV (normalized to 38 protons). The thermal distribution is clearly too narrow to fit the data. In the relativistic diffusion model, the data find their natural explanation in terms of nonequilibrium contributions (incomplete stopping) as described via the solution of the FPE. The transport coefficients can be found in table 1.

At this low energy, the ratio of the interaction time for central collisions to the rapidity relaxation time is about 1, indicating that the system is indeed not very far from equilibrium in rapidity space. Although this is also true at the slightly higher SIS-energies of 1.45 A GeV and 1.93 A GeV, the rapidity diffusion coefficients that allow to fit the data (fig.1) start to deviate from the values derived via the dissipation-fluctuation theorem by factors of 1.4 and 1.6, respectively, as illustrated in fig.2.

This deviation from the weak-coupling behaviour becomes more pronounced at AGS-energies. Here, the discrepancies between thermal distributions (dashed curves) and data are very dramatic, fig.3. The solutions of the FPE represent the experimental results quite accurately. In case of the light Si + Al system, the nonequilibrium character is particularly evident through the dip in the rapidity spectrum, which reflects the initial situation at t=0 and appears to persist even for central collisions. The rapidity diffusion coefficients that represent the data deviate from the values obtained via the dissipation-fluctuation theorem by factors of 3.3 and 3.7 for Si + Al and Au + Au, respectively (table 1).

This violation of the dissipation-fluctuation theorem becomes even larger at SPS-energies. Since the transport coefficients depend on the mass and mass asymmetry of the system as well as on the available energy ([2],[4]), systems of the same (or at least very similar) mass and mass asymmetry should be compared when investigating the dependence on incident energy. Data for the symmetric Au + Au system are available at SIS [7] - and AGS [6] - energies, whereas Pb + Pb - data [8] are used for the comparison at SPS - energies [2]. Here, eq.(2) is violated by a factor of 9 (corresponding to a factor of 3 in FWHM).

The results for the rapidity transport coefficients are shown in fig.4. The relaxation time drops monotonically from SIS to SPS energies (upper part). For the heavy Au + Au (or Pb + Pb) system, the values of the rapidity diffusion coefficient calculated from the dissipation-fluctuation theorem do not depend much on incident energy (lower part) because the inverse proportionality to the (dropping) relaxation time is compensated by the shrinking width of the corresponding thermal equilibrium distribution in Y-space with increasing incident rapidity $y_1$. However, the diffusion coefficients that are required to fit the data are seen to rise from SIS-energies, where the value from eq.(2) gives a reasonable representation of the data at 1.77 A GeV/c ($E_{beam}$ = 1.06 A GeV), via AGS- to SPS- energies, where clear violations of eq.(2) are observed.

This conclusion is not affected by the present uncertainties in the interaction times that enter the model calculation, because rescaled values of the interaction times (and correspondingly, of the rapidity relaxation time, keeping the ratio constant) would modify the diffusion coefficients according to eq.(2), but maintain the variances. Hence, the violation of the dissipation-fluctuation theorem in Y-space with increasing energy occurs independently of a detailed description of the dynamical behaviour, it is a consequence of the nonequilibrium-statistical properties of the system.

In transport descriptions of heavy-ion collisions at non-relativistic energies, one has tried to pin down a violation of the Einstein relations using various macroscopic variables (distance between the two center-of-mass, mass asymmetry, deformation). In spite of detailed theories [9], no compelling evidence was found for such a violation when comparing with data. For the rapidity variable at relativistic energies, however, the evidence is quite straightforward.

It would be interesting to investigate in more detail the origin of this effect in relativistic collisions. Whereas it could be due to the strong-coupling behaviour that leads to an increase of the diffusion coefficient above the values that I have obtained from eq.(2), a rapid collective expansion of the system (before thermal equilibrium is attained) also leads to broader distributions and hence, could cause an artificial enhancement in the diffusion coefficients that are used to represent the data. The enhancement might eventually also be due to non-equilibrium quark-gluon plasma formation prior to hadronization. Developing unambiguous tests to discriminate these possibilities is very challenging.

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Table I.

*Relaxation times $\tau_Y$ and rapidity diffusion coefficients $D_Y$ at SIS-, AGS- and SPS-energies. The incident momentum per particle is $p_L$, the temperature of the thermal equilibrium distribution [1,5,7] $T_p$. The fit values $D_Y$ of the diffusion coefficients exceed the values $D_Y^{th}$ obtained from the Einstein relation at the higher energies.*

| System | $p_L$ (GeV) | $y_1$ | $\tau_Y$ (fm) | $\tau_{int}/\tau_Y$ | $T_p$ (MeV) | $D_Y^{th}$ ($10^{-4}$fm$^{-1}$) | $D_Y$ ($10^{-4}$fm$^{-1}$) |
|---|---|---|---|---|---|---|---|
| Ni + Ni | 1.77 | 1.388 | 360 | 1.03 | 79 | 4.7 | 4.7 |
| | 2.20 | 1.586 | 310 | 1.02 | 84 | 4.4 | 6.2 |
| | 2.71 | 1.782 | 290 | 0.95 | 92 | 4.1 | 6.5 |
| Si + Al | 14.6 | 3.44 | 60 | 0.80 | 140 | 7.8 | 26.0 |
| Au + Au | 1.77 | 1.39 | 1100 | 1.14 | 80 | 1.6 | 1.6 |
| Au + Au | 11.4 | 3.19 | 370 | 1.08 | 130 | 1.4 | 5.1 |
| Pb + Pb | 157.7 | 5.83 | 134 | 0.81 | 160 | 1.4 | 12.4 |

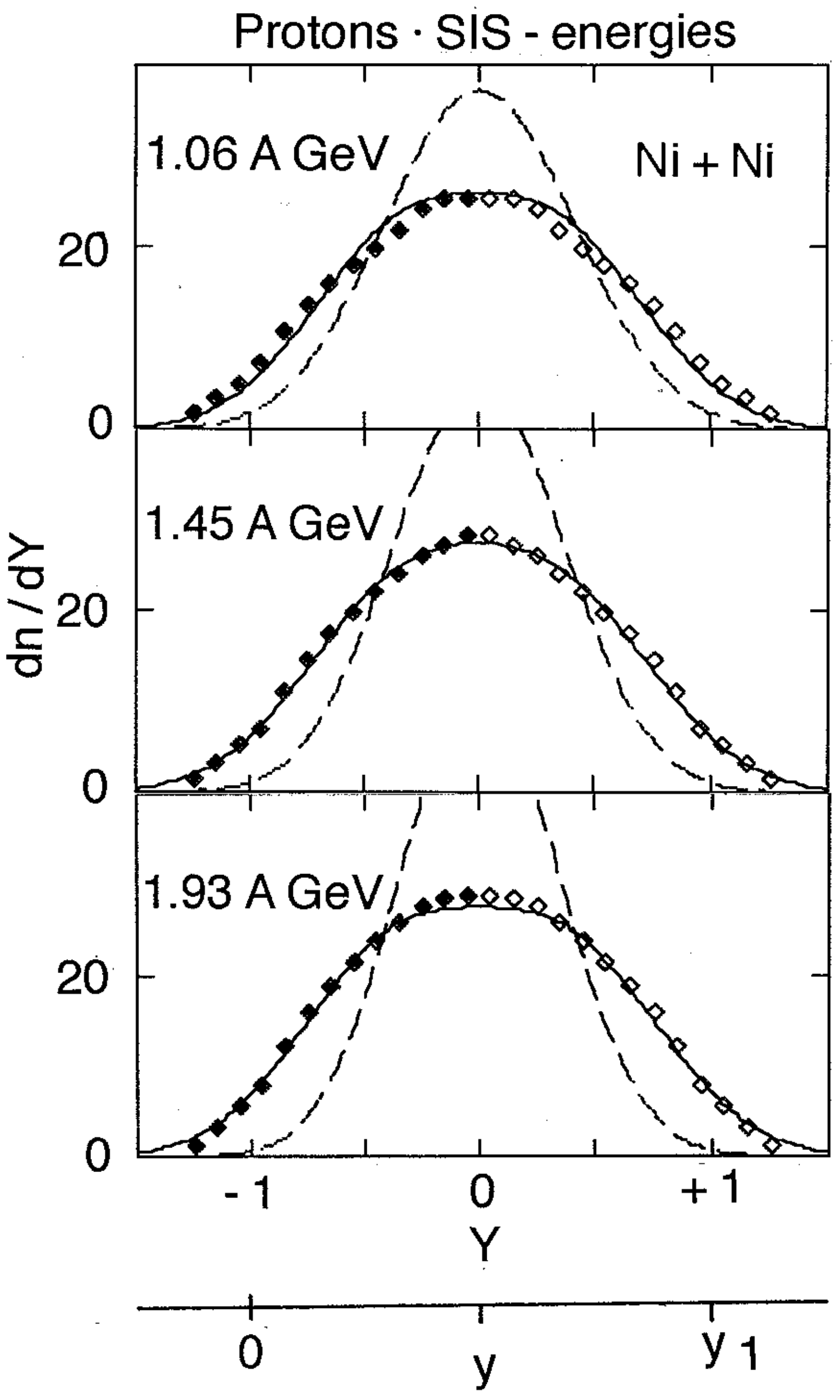
## Figure Captions

**FIG.1.** Rapidity relaxation of participant protons in central relativistic Ni + Ni- collisions at SIS- energies. FOPI data [5] are reflected about midrapidity. The dashed curves are thermal equilibrium distributions, solid curves are nonequilibrium results of the Relativistic Diffusion Model. At the lowest energy, the rapidity diffusion coefficient is obtained from the dissipation-fluctuation theorem, at the higher energies enhanced values are used to fit the data.

**FIG.2.** Dependence of the rapidity diffusion coefficients on incident momentum per particle in Ni + Ni reactions at SIS energies. The fit values deviate from the results of the dissipation-fluctuation theorem at the higher momenta. The lines are drawn to guide the eye.

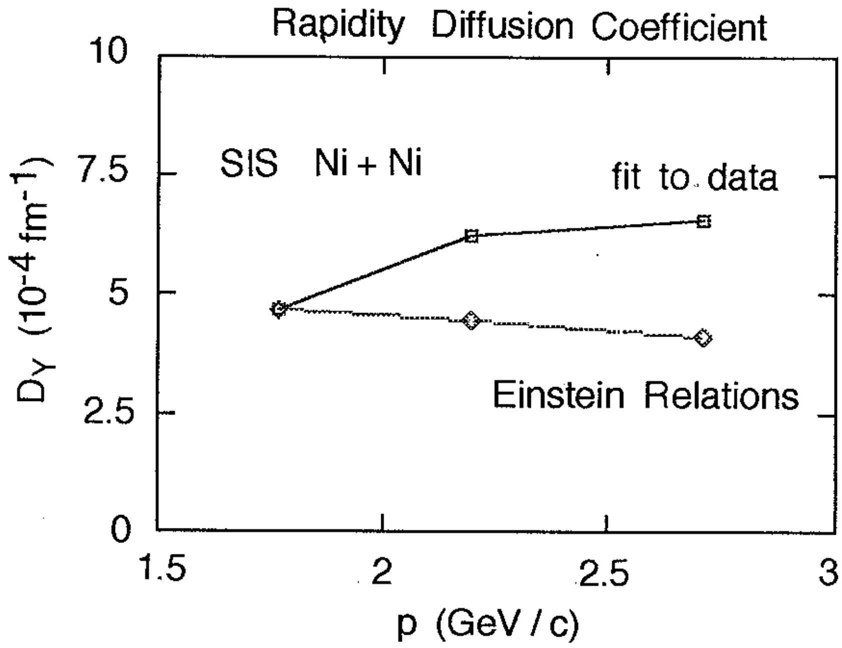
**FIG.3.** Rapidity relaxation of participant protons at AGS energies (cf. table 1). The Au + Au data and part of the Si + Al data [6] have been reflected about midrapidity. Isotropic thermal equilibrium distributions (dashed curves) are too narrow to fit the data. Agreement is achieved through the nonequilibrium solutions in the RDM, solid curves. The fit values of the diffusion coefficients exceed the ones obtained from eq.(2).

**FIG.4.** Dependence of the rapidity relaxation time and the rapidity diffusion coefficient on the incident momentum per particle in the energy-range from SIS via AGS to SPS. The values of the diffusion coefficients as derived from fits to Au + Au and Pb + Pb data are significantly larger than results from the dissipation-fluctuation theorem at the higher energies. Lines are drawn to guide the eye.



Protons · SIS - energies
1.06 A GeV
Ni + Ni
1.45 A GeV
1.93 A GeV
dn / dY
20
0
20
0
20
0
- 1
0
+ 1
Y
0
y
y 1


FIG. 1.



Rapidity Diffusion Coefficient
SIS Ni + Ni
fit to data
Einstein Relations
$D_Y$ ($10^{-4}$ fm$^{-1}$)
p (GeV / c)
10
7.5
5
2.5
0
1.5
2
2.5
3

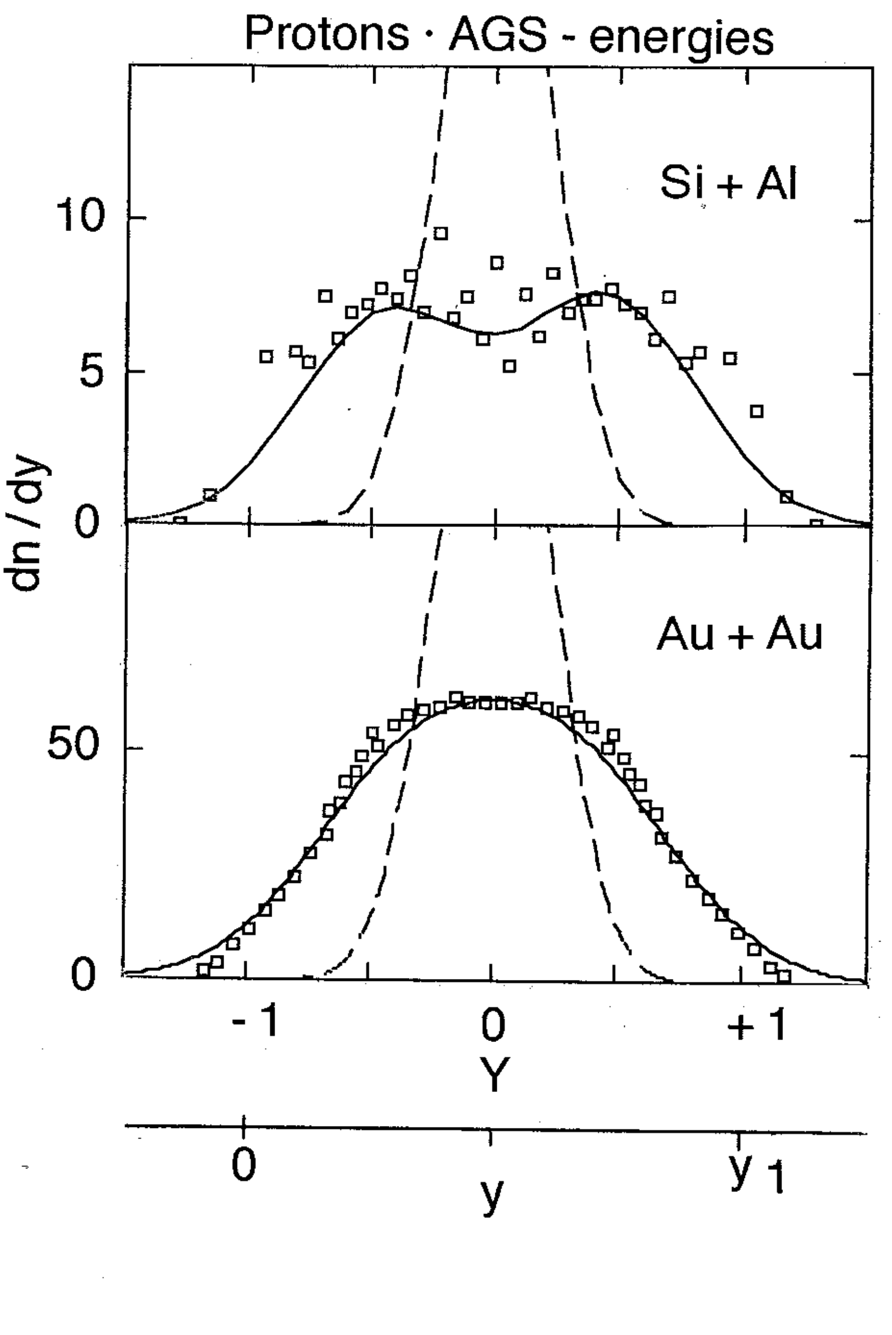
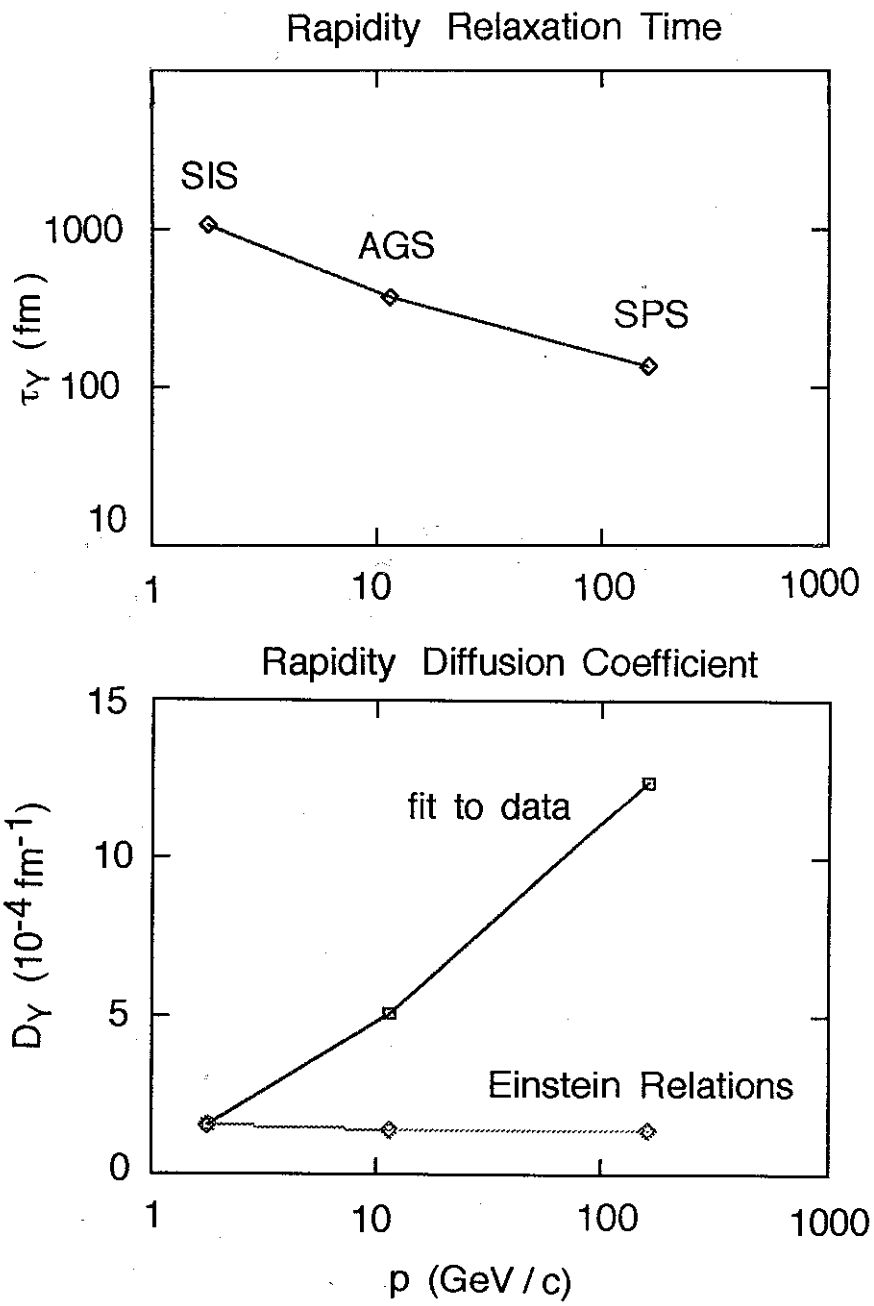



FIG. 3.



Rapidity Relaxation Time
SIS
AGS
SPS
$\tau_Y$ (fm)
1000
100
10
1
10
100
1000
Rapidity Diffusion Coefficient
fit to data
Einstein Relations
$D_Y$ ($10^{-4}$ fm$^{-1}$)
15
10
5
0
1
10
100
1000
p (GeV / c)